Scrambling Time from Local Perturbations of the Eternal BTZ Black Hole

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Density matrices

Pure state: vector in a Hilbert space

$$|\psi\rangle \in \mathcal{H}$$

Often quantum mechanics

$$|\psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Can we find $|\psi_A\rangle$ corresponding to $|\psi\rangle$?

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Can we find $|\psi_A\rangle$ corresponding to $|\psi\rangle$? **No**
Mixed state is probabilistic mixture of pure states:
state $|\Psi_i\rangle$ with probability p_i

Density matrix

$$\rho = \sum_n p_n |\Psi_n\rangle \langle\Psi_n|$$

Density matrices

$$\langle\psi|\mathcal{O}_A \otimes \mathbb{I}|\psi\rangle = \mathrm{Tr}(\mathcal{O}_A \rho_A)$$

Thermal states obey **Boltzmann** distribution

$$p_n = \frac{1}{Z(\beta)} e^{-\beta E_n}$$

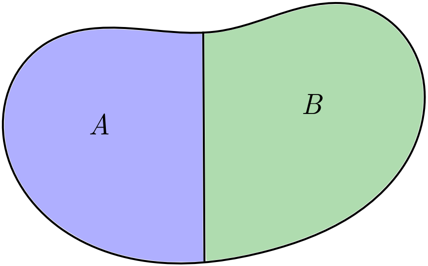
Thermal density matrix

$$\rho = \frac{1}{Z(\beta)} \sum_n e^{-\beta E_n} |\Psi_n\rangle \langle\Psi_n| = \frac{1}{Z(\beta)} e^{-\beta H}$$

Entanglement entropy

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Trace over one Hilbert space



Reduced density matrix

$$\rho_A = \mathrm{Tr}_B\, \rho$$

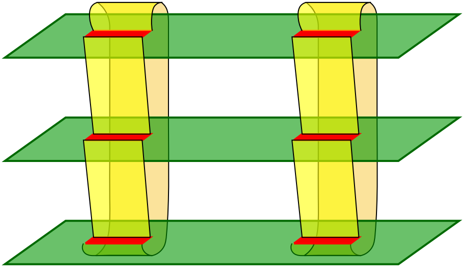
Entanglement entropy is von Neumann entropy of reduced density matrix

Entanglement entropy

$$S_A = -\,\mathrm{Tr}_A\, (\rho_A \log \rho_A)$$

Replica trick

- Calculate $S_n(A) = \frac{1}{1-n} \log \mathrm{Tr}_A(\rho_A^n)$ on $n-$sheeted Riemann surface

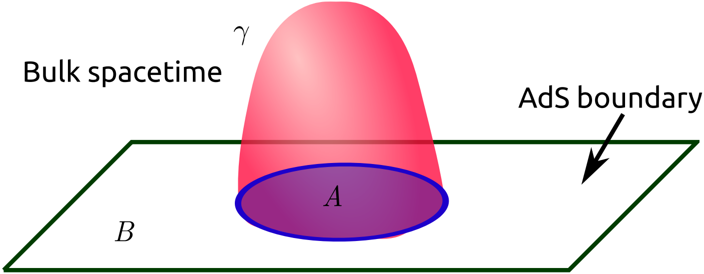


- Continue analytically
- $S_A = \lim_{n \to 1} S_n(A)$

Holographic entanglement entropy

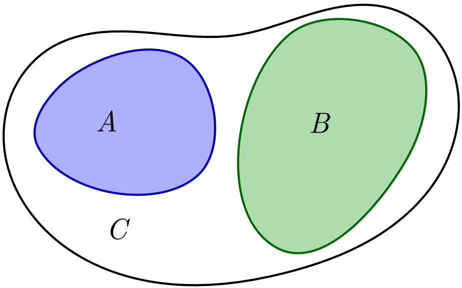
Ryu-Takayanagi formula

$$S_A = \min_{\partial\gamma=\partial A} \frac{\text{Area}(\gamma)}{4G_N}$$



Mutual information

Entangelment between two systems A and B



Mutual information

$$I_{A:B} = S_A + S_B - S_{A\cup B} \geq \frac{\left(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle\right)^2}{2\|\mathcal{O}_A\|^2 \|\mathcal{O}_B\|^2}$$

Provides an upper bound on correlators between A and B

Scrambling time

A natural question to ask is when correlators and entanglement get destroyed by the perturbation?

Scrambling time is defined by

$$I_{A:B}(t^{\star}) = 0$$

Our goal is to compute the **scrambling time** for thermofield double state

Thermofield double state (TFD)

Two non-interacting 2d CFTs with Hilbert spaces $\mathcal{H}_L \cong \mathcal{H}_R$
Choose a particular **entangled** state

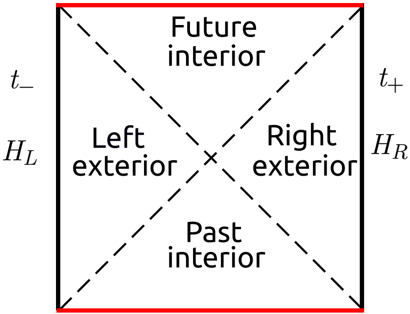
Thermofield double state in $\mathcal{H}_L \otimes \mathcal{H}_R$

$$|\Psi_\beta\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\frac{\beta}{2} E_n} |n\rangle_L |n\rangle_R$$

Tracing over $\mathcal{H}_L$ or $\mathcal{H}_R$ yields *thermal* reduced density matrix

Gravity dual description

Eternal BTZ BH is holographic dual of TFD



Two natural choices for Hamiltonian:

- $H = H_L \otimes \mathbb{I}_L - \mathbb{I}_R \otimes H_R$. $t_- = t_+ = t$
- $H = H_L \otimes \mathbb{I}_L + \mathbb{I}_R \otimes H_R$. $t_- = -t_+ = t$. Approximate description of two AdS BH

Adding a local perturbation

Perturb TFD by a primary operator ψ in the CFT$_L$ at time $-t_\omega$ in the past

$$\rho_L(t) = \mathcal{N} e^{-iH_L t} e^{-\epsilon H_L} \psi(0, -t_\omega) e^{2\epsilon H_L + \beta H_L} \psi^\dagger(0, -t_\omega) e^{-\epsilon H_L} e^{iH_L t}$$

Simpler in Euclidean time $(x + i\tau, x - i\tau)$

$$\mathcal{N} \psi(x_2, \bar{x}_2) e^{\beta H} \psi^\dagger(x_1, \bar{x}_1)$$

where

$$x_1 = t_- + t_\omega + i\epsilon, \qquad x_2 = t_- + t_\omega - i\epsilon$$
$$\bar{x}_1 = -t_- - t_\omega - i\epsilon, \qquad \bar{x}_2 = -t_- - t_\omega + i\epsilon$$

Thermal state $\Rightarrow \tau \sim \tau + \beta$

Analytic continuation in TFD

One-sided correlator

$$\langle \Psi_\beta | \mathcal{O}_L(x_1, 0) \mathcal{O}_L^\dagger(x_2, t) | \Psi_\beta \rangle$$
$$= \sum_{n,m} e^{-\beta E_n + it(E_n - E_m)} \langle n |_L \mathcal{O}_L(x_1, 0) | m \rangle_L \langle m |_L \mathcal{O}_L^\dagger(x_2, 0) | n \rangle_L$$

Two-sided correlator

$$\langle \Psi_\beta | \mathcal{O}_L(x_1, 0) \mathcal{O}_R(x_2, t) | \Psi_\beta \rangle$$
$$= \sum_{n,m} e^{-\beta E_n + i(t - i\frac{\beta}{2})(E_n - E_m)} \langle n |_L \mathcal{O}_L(x_1, 0) | m \rangle_L \langle m |_R \mathcal{O}_R^\dagger(x_2, 0) | n \rangle_R$$

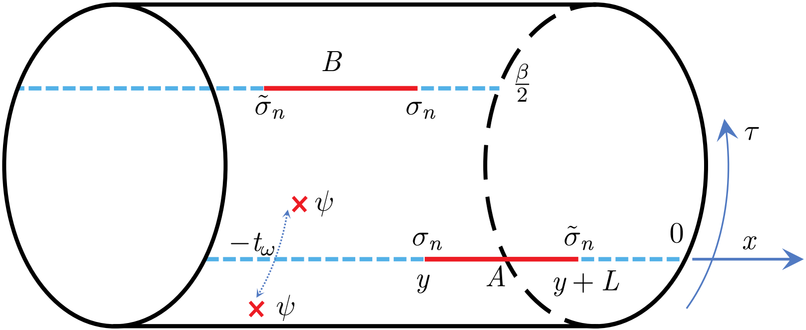
Analytic continuation in TFD

$$\langle \Psi_\beta | \mathcal{O}_L(x_1, 0) \mathcal{O}_L(x_2, t) | \Psi_\beta \rangle = \langle \Psi_\beta | \mathcal{O}_L(x_1, 0) \mathcal{O}_R^\dagger \left(x_2, t - i\frac{\beta}{2} \right) | \Psi_\beta \rangle$$

One-sided and two-sided correlators in TFD are related

$$t \mapsto t + i\frac{\beta}{2}$$

Setup for mutual information



Replica trick for S_A

Replica trick $\to$ replicate cylinder n times, glue along the cuts
Instead, twist operators can be used

$$\mathrm{Tr}\,\rho_A^n(t) = \frac{\langle \Psi(x_1,\bar{x}_1)\sigma_n(x_2,\bar{x}_2)\tilde{\sigma}_n(x_3,\bar{x}_3)\Psi^\dagger(x_4,\bar{x}_4)\rangle_{C_n}}{\left(\langle \psi(x_1,\bar{x}_1)\psi^\dagger(x_4,\bar{x}_4)\rangle_{C_1}\right)^n}$$

$$\begin{aligned} x_1 &= -i\epsilon, \quad & x_2 &= y - t_\omega - t_-, \quad & x_3 &= y + L - t_\omega - t_-, \quad & x_4 &= +i\epsilon \\ \bar{x}_1 &= +i\epsilon, \quad & \bar{x}_2 &= y + t_\omega + t_-, \quad & \bar{x}_3 &= y + L + t_\omega + t_-, \quad & \bar{x}_4 &= -i\epsilon \end{aligned}$$

$$\Psi = \psi_1 \cdot \psi_2 \cdot \ldots \cdot \psi_n$$

Ψ has a ψ_i for each copy of the theory
Conformal dimension nh_ψ
Conformal dimension of twist operators is $2H_\sigma$

$$H_\sigma = \frac{c}{24}\left(n - \frac{1}{n}\right)$$

Correlators on the plane

We are in a thermal state on the cylinder

Map to the plane

$$w(x) = e^{\frac{2\pi}{\beta} x}$$

- 2-pt correlators are fixed by conformal symmetry
- 4-pt correlators are expressed via conformal blocks $G(z, \bar{z})$

$$\mathrm{Tr}\, \rho_A^n(t) = \left| \frac{\beta}{\pi \varepsilon_{UV}} \sinh\left(\frac{\pi(x_2 - x_3)}{\beta} \right) \right|^{-4H_\sigma} |1 - z|^{4H_\sigma}\, G(z, \bar{z})$$

where

$$z = \frac{(w_1 - w_2)(w_3 - w_4)}{(w_1 - w_3)(w_2 - w_4)}$$

Approximation: $z = 1 + f(y, L, \beta, t)\epsilon + \mathcal{O}(\epsilon^2)$

Evaluating $G(z, \bar{z})$

4-pt function $\langle \Psi(x_1, \bar{x}_1)\sigma_n(x_2, \bar{x}_2)\tilde{\sigma}_n(x_3, \bar{x}_3)\Psi^\dagger(x_4, \bar{x}_4)\rangle_{C_n}$
contains two light and two heavy operators
Fitzpatrick, Kaplan and Walters considered such setup in
arXiv:1403.6829

$$\log G(z, \bar{z}) \simeq -\frac{c(n-1)}{6} \log \frac{z^{\frac{1}{2}(1-\alpha_\psi)} \bar{z}^{\frac{1}{2}(1-\bar{\alpha}_\psi)}(1-z^{\alpha_\psi})(1-\bar{z}^{\bar{\alpha}_\psi})}{\alpha_\psi \bar{\alpha}_\psi}$$

where

$$\alpha_\psi = \sqrt{1 - \frac{24 h_\psi}{c}} \,,$$

Entanglement entropy S_A

$$S_A = S_{\text{thermal}} + \Delta S_A$$

Thermal part of entanglement entropy is

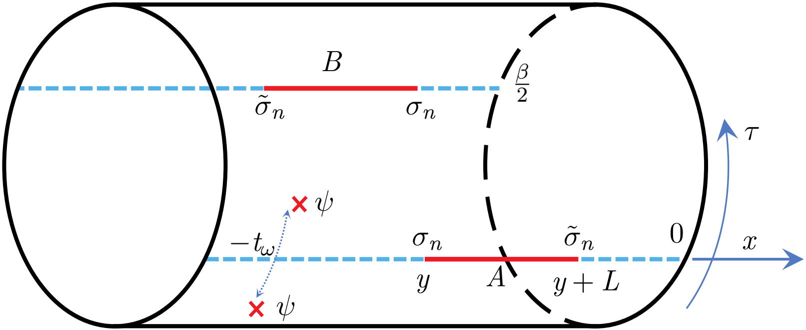
$$S_{\text{thermal}} = \frac{c}{3}\log\left(\frac{\beta}{\pi\varepsilon_{UV}}\sinh\frac{\pi L}{\beta}\right)$$

ΔS_A, entanglement entropy due to perturbation

$$\begin{cases} 0, & t_- + t_\omega < y \\ \frac{c}{6}\log\left[\frac{\beta}{\pi\epsilon}\frac{\sin\pi\alpha_\psi}{\alpha_\psi}\frac{\sinh\left(\frac{\pi(y+L-t_- - t_\omega)}{\beta}\right)\sinh\left(\frac{\pi(t_- + t_\omega - y)}{\beta}\right)}{\sinh\left(\frac{\pi L}{\beta}\right)}\right] \\ 0, & t_- + t_\omega > y + L \end{cases}$$

No change before perturbations arrives or after it leaves A

Setup for mutual information



Entanglement entropy S_B

$$\mathrm{Tr}\,\rho_B^n(t) = \frac{\langle \Psi(x_1,\bar{x}_1)\sigma_n(x_5,\bar{x}_5)\tilde{\sigma}_n(x_6,\bar{x}_6)\Psi^\dagger(x_4,\bar{x}_4)\rangle_{C_n}}{\left(\langle \psi(x_1,\bar{x}_1)\psi^\dagger(x_4,\bar{x}_4)\rangle_{C_1}\right)^n}$$

$$x_5 = y + L + i\frac{\beta}{2} - t_\omega - t_+, \quad x_6 = y + i\frac{\beta}{2} - t_\omega - t_+$$
$$\bar{x}_5 = y + L - i\frac{\beta}{2} + t_\omega + t_+, \quad \bar{x}_6 = y - i\frac{\beta}{2} + t_\omega + t_+$$

Entanglement entropy remains thermal

$$S_B = \frac{c}{3}\log\left(\frac{\beta}{\pi\varepsilon_{UV}}\sinh\frac{\pi L}{\beta}\right)$$

$A \cup B$

$$\mathrm{Tr}\,\rho^n_{A\cup B} = \frac{\langle \psi(x_1,\bar{x}_1)\sigma_n(x_2,\bar{x}_2)\tilde{\sigma}_n(x_3,\bar{x}_3)\sigma_n(x_5,\bar{x}_5)\tilde{\sigma}_n(x_6,\bar{x}_6)\psi^\dagger(x_4,\bar{x}_4)\rangle}{\left(\langle \psi(x_1,\bar{x}_1)\psi^\dagger(x_4,\bar{x}_4)\rangle_{C_1}\right)^n}$$

Map to the plane $w(x) = \exp\left(\frac{2\pi}{\beta}x\right)$ and

$$z(w) = \frac{(w_1 - w)(w_3 - w_4)}{(w_1 - w_3)(w - w_4)}$$

We obtain the following 6-pt function

$$\langle \psi | \sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)\sigma_n(z_5,\bar{z}_5)\tilde{\sigma}_n(z_6,\bar{z}_6)|\psi\rangle$$

S and T-channels: Resolution of Identity

S-channel

$$\langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)\sigma_n(z_5,\bar{z}_5)\tilde{\sigma}_n(z_6,\bar{z}_6)|\psi\rangle$$
$$= \sum_\alpha \langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)|\alpha\rangle \langle\alpha|\sigma_n(z_5,\bar{z}_5)\tilde{\sigma}_n(z_6,\bar{z}_6)|\psi\rangle$$

T-channel

$$\langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)\sigma_n(z_5,\bar{z}_5)\tilde{\sigma}_n(z_6,\bar{z}_6)|\psi\rangle$$
$$= \sum_\alpha \langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(z_6,\bar{z}_6)|\alpha\rangle \langle\alpha|\sigma_n(z_5,\bar{z}_5)\tilde{\sigma}_n(1,1)|\psi\rangle$$

S-channel

OPE of twist operators

$$\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1) \sim \mathbb{I} + \mathcal{O}\left((z-1)^r\right) \qquad r \in \mathbb{N}$$

Orthogonality of 2-pt functions

$$\sum_\alpha \langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)|\alpha\rangle \langle\alpha| \simeq \langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)|\psi\rangle \langle\psi|$$

So the first 4-pt function is the same as in S_A while the second the same as in S_B

$S_{A\cup B} = S_A + S_B$ and $I_{A:B} = 0$

T-channel

Again, dominant contribution comes from $|\alpha\rangle = |\psi\rangle$.

$$\langle\psi|\sigma_n(z_5,\bar{z}_5)\tilde{\sigma}_n(1,1)|\psi\rangle = G(z_5,\bar{z}_5)$$
$$\langle\psi|\sigma_n(z,\bar{z})\tilde{\sigma}_n(z_6,\bar{z}_6)|\psi\rangle = |1-\tilde{z}_2|^{4H_\sigma}|z_2-z_6|^{-4H_\sigma}G(\tilde{z}_2,\bar{\tilde{z}}_2)$$

Cross-ratios are not the same as in S-channel

Cross-ratios

$$z_5 = 1 - \frac{2\pi i\epsilon}{\beta} \frac{\cosh \frac{\pi(t_- - t_+)}{\beta}}{\sinh \frac{\pi(y+L-t_- - t_\omega)}{\beta} \cosh \frac{\pi(y+L-t_+ - t_\omega)}{\beta}} + \mathcal{O}(\epsilon^2)$$

$$\bar{z}_5 = 1 + \frac{2\pi i\epsilon}{\beta} \frac{\cosh \frac{\pi(t_- - t_+)}{\beta}}{\sinh \frac{\pi(y+L+t_- + t_\omega)}{\beta} \cosh \frac{\pi(y+L+t_+ + t_\omega)}{\beta}} + \mathcal{O}(\epsilon^2)$$

$$\tilde{z}_2 = 1 + \frac{2\pi i\epsilon}{\beta} \frac{\cosh \frac{\pi(t_- - t_+)}{\beta}}{\sinh \frac{\pi(y-t_- - t_\omega)}{\beta} \cosh \frac{\pi(y-t_+ - t_\omega)}{\beta}} + \mathcal{O}(\epsilon^2)$$

$$\tilde{\bar{z}}_2 = 1 - \frac{2\pi i\epsilon}{\beta} \frac{\cosh \frac{\pi(t_- - t_+)}{\beta}}{\sinh \frac{\pi(y+t_- + t_\omega)}{\beta} \cosh \frac{\pi(y+t_+ + t_\omega)}{\beta}} + \mathcal{O}(\epsilon^2)$$

Entanglement entropy $S_{A\cup B}$

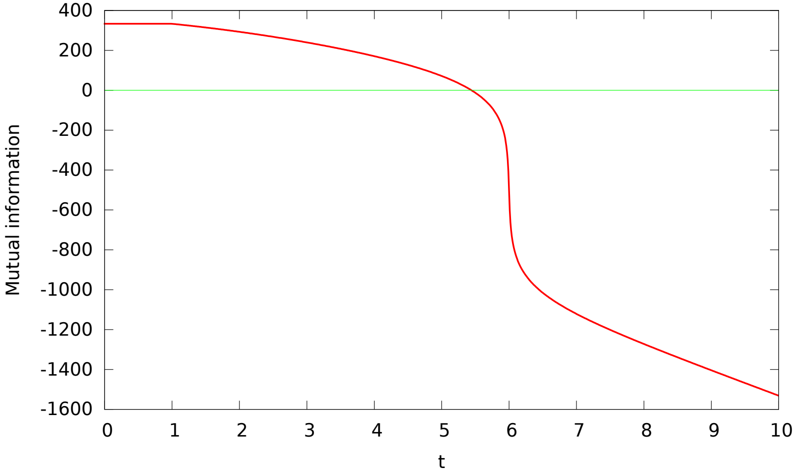
$$S_{A\cup B} \simeq \frac{2c}{3}\log\left|\frac{\beta}{\pi\varepsilon_{UV}}\cosh\left(\frac{\pi\Delta t}{\beta}\right)\right| \qquad t_- + t_\omega < y$$

$$S_{A\cup B} \simeq \frac{2c}{3}\log\left|\frac{\beta}{\pi\varepsilon_{UV}}\cosh\left(\frac{\pi\Delta t}{\beta}\right)\right|$$
$$+\frac{c}{6}\log\left(\frac{\beta}{\pi\epsilon}\frac{\sin\pi\alpha_\psi}{\alpha_\psi}\frac{\sinh\frac{\pi(t_-+t_w-y)}{\beta}\cosh\frac{\pi(t_++t_w-y)}{\beta}}{\cosh\frac{\pi\Delta t}{\beta}}\right)$$
$$y < t_- + t_\omega < y + L$$

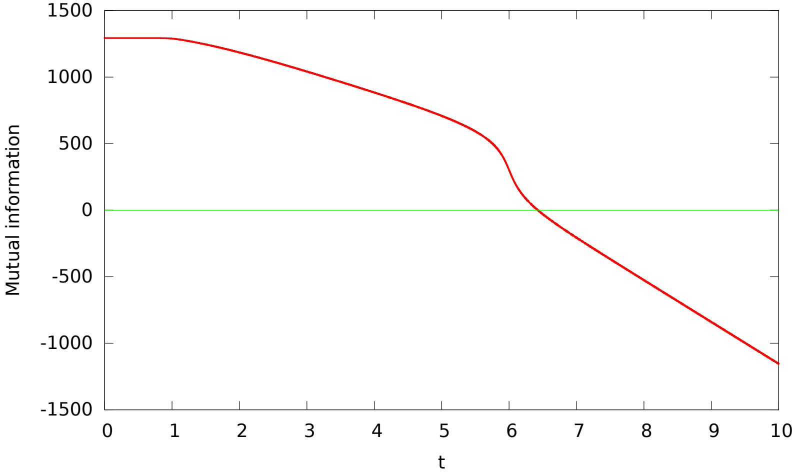
Entanglement entropy $S_{A\cup B}$

$$
\begin{aligned}
S_{A\cup B} \simeq\ & \frac{2c}{3}\log\left|\frac{\beta}{\pi\varepsilon_{UV}}\cosh\left(\frac{\pi\Delta t}{\beta}\right)\right| + \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\frac{\sin\pi\alpha_\psi}{\alpha_\psi}\right)\\
&+\frac{c}{6}\log\left(\frac{\sinh\frac{\pi(t_-+t_\omega-y)}{\beta}\cosh\frac{\pi(t_++t_\omega-y)}{\beta}}{\cosh\frac{\pi\Delta t}{\beta}}\right)\\
&+\frac{c}{6}\log\left(\frac{\sinh\frac{\pi(t_-+t_\omega-y-L)}{\beta}\cosh\frac{\pi(t_++t_\omega-y-L)}{\beta}}{\cosh\frac{\pi\Delta t}{\beta}}\right)\\
&t_- + t_\omega > y + L
\end{aligned}
$$

Mutual information



Mutual information



Scrambling time

Scrambling time is defined by

$$I_{A:B}(t_\omega^\star) = 0$$

Suppose $t_\omega > y + L$ and $\dfrac{t_\omega^\star}{\beta} \gg 1$

Scrambling time in perturbed TFD

$$t_\omega^\star = y + \frac{L}{2} - \frac{\beta}{2\pi}\log\left(\frac{\beta}{\pi\epsilon}\frac{\sin\pi\alpha_\psi}{\alpha_\psi}\right) + \frac{\beta}{\pi}\log\left(2\sinh\frac{\pi L}{\beta}\right)$$

Holographic description

- Gravity dual of TFD is eternal BTZ black hole
- Approximate local perturbation of TFD by a **free falling particle** in the BTZ black hole
- Compute back-reaction of particle on metric of BTZ:
  - Map BTZ Kruskal coordinates to AdS_3
  - Back-reaction in AdS_3 is known: conical singularity or BTZ black hole
  - (Map back-reacted metric back to original coordinates)

Free falling particle

AdS-Schwarzschild patch of the BTZ black hole

$$ds^2 = \frac{R^2}{z^2}\left[-\left(1 - Mz^2\right) dt_-^2 + \frac{dz^2}{1 - Mz^2} + d\theta^2\right], \qquad \theta \sim \theta + 2\pi$$

The geodesic of a free falling particle satisfying $z(-t_\omega) = \epsilon$

$$t_- = \tilde{\tau}, \quad \theta = 0, \quad 1 - Mz^2 = (1 - M\epsilon^2)\cosh^{-2}\left(\sqrt{M}(\tilde{\tau} + t_\omega)\right)$$

Energy of the particle

$$E = \frac{m\,R}{\epsilon}\sqrt{1 - M\epsilon^2}$$

matches energy of CFT perturbation if

$$m = \frac{2h_\psi}{R}$$

Free falling particle in Kruskal coordinates

Metric in Kruskal coordinates

$$ds^2 = R^2 \frac{-4dudv + (-1+uv)^2 d\phi^2}{(1+uv)^2}$$

Action of the particle

$$S = -2mR \int \frac{\sqrt{v'}\,du}{1+uv}$$

$$v(u) = \frac{C_2 + (C_1 + C_2^2)u}{1 + C_2 u}$$

where

$$C_1 = M\epsilon^2 e^{2\sqrt{M}t_\omega}$$

$$C_2 = -\sqrt{1 - M\epsilon^2}\, e^{\sqrt{M}t_\omega}$$

Map to AdS_3

Map to AdS_3 maps dynamic particle to static particle, so we also need to apply boosts

$$r = \frac{R}{\sqrt{M}\epsilon}\left|\frac{1-uv}{1+uv}\right| \cdot \sqrt{M\epsilon^2 \sinh^2\phi + \left(\frac{e^{\sqrt{M}t_\omega}\, u - e^{-\sqrt{M}t_\omega}\, v}{1-uv} - \sqrt{1-M\epsilon^2}\cosh\phi\right)^2}$$

$$\tan\tau = \sqrt{M}\epsilon \frac{\frac{e^{\sqrt{M}t_\omega}\, u + e^{-\sqrt{M}t_\omega}\, v}{1-uv}}{\cosh\phi - \sqrt{1-M\epsilon^2}\,\frac{e^{\sqrt{M}t_\omega}\, u - e^{-\sqrt{M}t_\omega}\, v}{1-uv}}$$

$$\tan\varphi = \sqrt{M}\epsilon \frac{\sinh\phi}{\frac{e^{\sqrt{M}t_\omega}\, u - e^{-\sqrt{M}t_\omega}\, v}{1-uv} - \sqrt{1-M\epsilon^2}\cosh\phi}$$

Back-reaction in AdS$_3$

We mapped point particle to $r = 0$, $\varphi = 0$ in AdS$_3$ coordinates

$$ds^2 = -\left(r^2 + R^2\right) d\tau^2 + \frac{R^2\, dr^2}{r^2 + R^2} + r^2\, d\varphi^2$$

Back-reaction in AdS$_3$ is known:

$$ds^2 = -\left(r^2 + R^2 - \mu\right) d\tau^2 + \frac{R^2\, dr^2}{r^2 + R^2 - \mu} + r^2\, d\varphi^2$$

$\mu = 8\, G_N R^2 m$ is related to mass of the particle m

Length of geodesics

Length of *spacelike* geodesics in locally AdS_3 space is known
Let X be region with endpoints $(r_\infty^{(1)}, \tau_\infty^{(1)}, \varphi_\infty^{(1)})$ and $(r_\infty^{(2)}, \tau_\infty^{(2)}, \varphi_\infty^{(2)})$

$$S_X = \frac{c}{6} \log \left[\frac{2 r_\infty^{(1)} \cdot r_\infty^{(2)}}{R^2} \frac{\cos\left(|\Delta\tau_\infty| a\right) - \cos\left(|\Delta\varphi_\infty| a\right)}{a^2} \right]$$

where

$$a \equiv \sqrt{1 - \frac{\mu}{R^2}} = \alpha_\psi$$

carries information about perturbation

Holographic entanglement entropy

Pick region A to be an interval with endpoints

$$(t_-, z_\infty, y) \qquad \text{and} \qquad (t_-, z_\infty, y + L)$$

Region B:

$$(t_+, z_\infty, y) \qquad \text{and} \qquad (t_+, z_\infty, y + L)$$

S_A and S_B match CFT result

Holographic entanglement entropy

For $A \cup B$ we have two cases: corresponds to S and T-channels

- Endpoints on the same boundary are connected
- Endpoints on different boundaries are connected